

The Birth and Death Process Enhanced Optimization Algorithm for Multi-Access Selection in Heterogeneous Wireless Networks*

SHUO ZHANG AND QI ZHU

*State Key Laboratory of Software Development Environment
Jiangsu Key Lab of Wireless Communications, Key Lab on Wideband Wireless
Communications and Sensor Network Technology of Ministry of Education
Nanjing University of Posts and Telecommunications
Jiangsu Nanjing, 21000 P.R. China
E-mail: zhuqi@njupt.edu.cn*

To further enhance the system performance of the heterogeneous network selection algorithm, a Markov process based utility optimization method for multi-access selection in heterogeneous wireless networks is presented in this paper. The method adopts the birth and death process to analyze the system performance of heterogeneous networks, fully considers users' mobility and then obtains network joint utility optimization model. To maximize the joint utility, the proportions of different multi-access selections in heterogeneous wireless network overlapping region should be well organized. The subgradient method was adopted to obtain the best proportions for solving this optimization problem. The simulation results show that the method can maximize system utility, effectively improve throughput and make the radio resource allocation more efficient.

Keywords: heterogeneous networks, multi-access selection, birth and death process, joint utility optimization, subgradient method

1. INTRODUCTION

With large-scale commercial 4G technology, the selection of wireless network services became more various for users. The development of next generation wireless network technology makes the situation of coexistence of a variety of heterogeneous networks more complicated. As an efficient RRM (Radio Resource Management) technology, heterogeneous network integration has been widely studied because it can use a variety of heterogeneous networks effectively. Heterogeneous network selection algorithms are always regarded as the key technology to realize network convergence and therefore became a research hotspot.

CRRM (Common Radio Resource Management), put forward by the 3GPP organization, can effectively manage radio resources, and its structure and model are given in literature [1]. Researchers have made some progress on heterogeneous network access selection methods after several years of effort. Literature [2] puts forth a fuzzy neural theory enhanced access selection algorithm in heterogeneous wireless networks. Literature [3] proposes an evolutionary game theory based network selection method to im-

Received September 12, 2014; revised December 22, 2014 & February 9, 2015; accepted March 3, 2015.
Communicated by Meng Chang Chen.

* This work is supported by National Natural Science Foundation of China (61571234), 973 program: 2013CB-329005 and 863 Program (2014AA01A705).

prove energy efficiency. Literature [4] gives a network selection algorithm based on AHP (Analytic Hierarchy Process) and TOPSIS (Total Order Preference by Similarity to the Ideal Solution). In literature [5], an end-to-end wireless resource management method, based on different traffic classes, is proposed. The method takes users' mobility as an important factor affecting network selection. In literature [6], the method guided by user preferences combines EW (Entropy Weight) method and FAHP (Fuzzy Analytic Hierarchy Process) to obtain a satisfying QoE (Quality of Experience). Under the precondition of system total bandwidth constraint, literature [7] converts a number of objective functions into constraints and chooses a reasonable heterogeneous network to maximize the time used by terminals in the network. However all of the heterogeneous network selection targets of the above methods are single networks. The advanced development of terminal technology makes it possible for a terminal to access several radio networks simultaneously. With the complicated situation of heterogeneous network coexistence, a multi-access selection algorithm has a great advantage at optimizing system performance, enhancing QoS (Quality of Service) and improving the efficiency of radio resources. Literature [8] designs a multi-access selection mechanism for multi-mode terminals which exploits the multi-radio transmission diversity of the heterogeneous wireless networks for accessing multiple networks simultaneously and executing parallel transmission. The method only addresses the selection problem for a single mobile terminal. In Literature [9], a multi-access selection algorithm based on joint utility optimization in heterogeneous wireless networks is presented for multiple mobile terminals. Lagrange multiplier and dual optimization method are used for utility optimization. However the utility only includes the bandwidth and cost. Blocking rate, which is very important for the multi-access selection method, is ignored. Additionally the results of Lagrange multiplier and dual optimization method are approximate, not really maximizing the joint utility. Kemeng Yang presents a context-aware vertical soft handoff algorithm (CAVSH) for heterogeneous wireless networks in [10]. In its active set selection process of CAVSH, the network selection is divided into several sub-problems without considering from the perspective of the whole system.

Considering the previously mentioned drawbacks, a Markov process enhanced utility optimization algorithm for multi-access selection in heterogeneous wireless networks is proposed in this paper. The concept of the birth and death process is used to analyze the system performance of heterogeneous networks and to obtain a Markov steady-state distribution of each network. A joint utility of the heterogeneous network system is derived by further derivation. In this paper, the utility includes the total unit-price throughput and network access probability. Then we can get the objective optimization function. The objective function contains a rate factor vector expressing the proportions of users selecting a different single heterogeneous network and all the networks in the overlapping area. A reasonable vector can maximize the system objective function. In this paper, we adopt the subgradient method to get the optimal rate vector of this objective function. The simulation results show that the method can maximize the system utility, effectively improve throughput and make the radio resource allocation more efficient.

The following contents are arranged as follows: Section 2 gives the heterogeneous network system model, handoff rate and the theory of the Markov process; Section 3 constructs a system joint utility based on the Markov steady-state distribution and adopts the subgradient method to solve the multi-access optimization problem; In section 4, the simu-

lation results show that the method can make better use of radio resources, maximize the joint utility and improve the throughput. Finally we conclude the study in section 5.

2. SYSTEM MODEL OF HETEROGENEOUS NETWORKS AND STEADY-STATE DISTRIBUTION

The heterogeneous network system model of this paper is shown in Fig. 1, which is similar to that in literature [11]. The heterogeneous scenario consists of two RANs (Radio Access Networks), and the two networks have some overlapping areas. RRM entities corresponding to each network are responsible for collecting network parameter information, including terminal mobility speed, network coverage radius, bandwidth of each channel, the number of channels and the traffic arrival rate, and then sending them to the CRRM servers. A CRRM server conducts the network selection decision-making according to the received information and sends them to an RRM entity as feedback. Normally the RRM entity will be responsible for the decision execution and resource allocation.

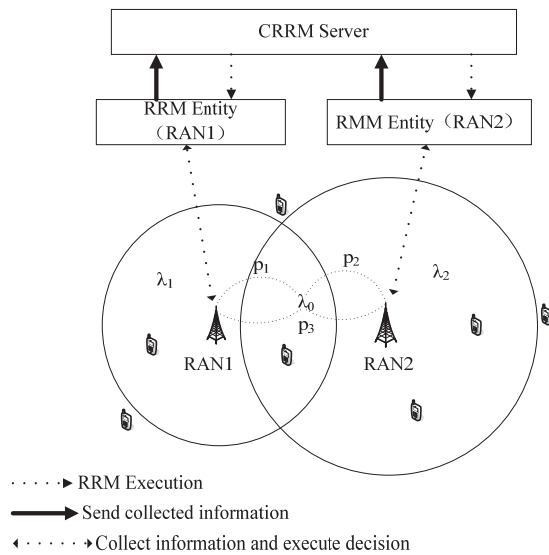


Fig. 1. System model of heterogeneous and CRRM structure.

For convenience, we assume that the traffic class is data traffic. The network parameters are presented as following: The arrival rates obey the Poisson Distribution, λ_0 is the traffic arrival rate in the overlapping area and the traffic arrival rates of RAN1 and RAN2 in the non-overlapping area are λ_1 and λ_2 respectively. In the system model of this paper, λ_0 is dependent on the overlapping area and λ_1 and λ_2 are dependent on the area of non-overlap area in RAN1 and RAN2. That is to say, if the overlapping area decreases, correspondingly, λ_0 should also decrease and λ_1 and λ_2 should increase to keep the average traffic rate; The service time, which means connection time, obeys a negative exponential distribution and its mean is defined as $1/\mu$, ν presents the average

terminal speed; d_1 and d_2 are defined as the coverage radius of RAN1 and RAN2, and the numbers of the assigned service channels of RAN1 and RAN2 are C_1 and C_2 respectively. Additionally the Poisson distribution and the negative exponential distribution have the memory-free property, which means that the current and future situation is only related to the current situation and has nothing to do with the previous one [12]. To make good use of the Markov theory, we rule that every channel provided by a given network has the same data rate and we define R_1 and R_2 for channels belonging to RAN1 and RAN2. Specifically the costs of using the two kinds of channels are $Cost_1$ and $Cost_2$ respectively.

This paper takes users' mobility as an important factor affecting a system's performance, and users' mobility is embodied by handoff rate. Literature [13] gives the expression of handoff rate based on an angle mobility model. The calculation method is used well in our model, and the expression is

$$\lambda^h = \frac{2nv}{\pi d}, \quad (1)$$

where n denotes the number of users, d represents the coverage radius of cell and v is average speed of terminals.

Because of the abundance of mobile terminal technology, it is assumed that terminals in this heterogeneous network system are all multi-homed terminals, which are different from multi-mode terminals. A multi-homed terminal supports parallel transmission of mobile data, which means that the terminal can access different heterogeneous networks simultaneously with different IPs, thereby enjoying different network services. Access to several heterogeneous networks simultaneously supports users' mobility better and therefore can enhance the system performance [14]. Fig. 2 presents a multi-access model of multi-homed terminals carrying data communication in an overlapping area of heterogeneous networks. According to this model, RRM entities collect parameter information and then send them to CRRM servers for decision-making. RRM entities will conduct resource allocation according to the decision information received from CRRM servers. Specifically they will inform each terminal of access information, permitting them to access to RAN1, RAN2 or both of the two networks. From the terminals' side, they can choose whether to access or not. Consequently both the requirement of utility maximum and users' requests will be fully met.

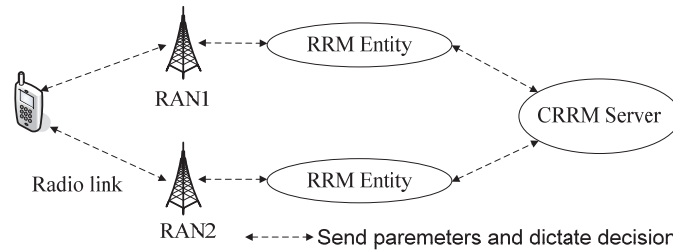


Fig. 2. Multi-access model of multi-homed terminals in overlapping area.

This paper adopts a continuous time Markov chain – the birth and death process to analyze the performance of each network [12]. Assuming that the number of channels of a network is C , each user can only take up a channel in a network and therefore the

maximum number of users in a network is C . Considering the situation that no channel is taken up in a network, there are $C+1$ network states in total, and the Markov process is shown in Fig. 3. In this figure, i means there are i channels are taken up now. Besides λ^i and μ^i the birth rate and death rate in state i . According to literature [12], the steady-state distribution is

$$\pi^i = \left(1 + \sum_{m=1}^C \left(\prod_{k=0}^{m-1} \lambda^k / \prod_{l=1}^m \mu^l \right) \right)^{-1} \left(\prod_{k=0}^{i-1} \lambda^k / \prod_{l=1}^i \mu^l \right), 0 \leq i \leq C. \quad (2)$$

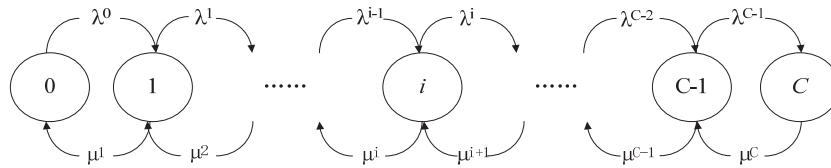


Fig. 3. Markov chain.

3. THE BIRTH AND DEATH PROCESS ENHANCED OPTIMIZATION ALGORITHM FOR MULTI-ACCESS SELECTION IN HETEROGENEOUS WIRELESS NETWORKS

3.1 System Utility Function

In the overlapping area of the two networks shown in Fig. 1, mobile users have the opportunity to access different heterogeneous networks. Reasonable network selections that meet users' demand will obtain a satisfying performance, such as large throughput and high network access rate. This method will calculate an optimal proportion vector to allocate radio resource effectively and efficiently. According to Fig. 1, it is ruled that there are three options for users in the overlapping region, namely access to RAN1, RAN2 and access to both of the two networks simultaneously. It is defined that the proportions of traffic arrival rate of the three cases out of total traffic arrival rate in the overlapping area are p_1 , p_2 and p_3 respectively. Based on current network resources, the CRRM server will calculate a reasonable proportion vector, namely $\bar{p} = (p_1, p_2, p_3)$, and send it to the RRM entity for resource management.

We use the birth and death process to analyze network performance in this method. According to Fig. 3, suppose that the RANs in the heterogeneous network scenario are RAN1 and RAN2, there are C_j+1 Markov states in network $j(j = 1, 2)$. The birth rate from state i to $i+1$ is defined as λ^i . The proportion of traffic arrival rate of access to network j in the overlapping area is p_j and the rate of access to both networks is p_3 . Therefore the total traffic arrival rate of network j in overlapping area is $(p_j + p_3)\lambda_0$. Apart from traffic arrival rate of the whole cell, the birth rate should also include average the handoff rate because of users' mobility. According to the theory of dynamic balance in literature [13], the number of users switching out from network j equals that of external terminals switching into this network. On account of this, here we calculate the probability of us-

er's switching out of network j instead of the average handoff rate, which is defined as $\bar{\lambda}_j^h$. On the condition of the steady state, we assume that the average number of users in network j is M_j . According to formula (1), the expression of average handoff rate is

$$\bar{\lambda}_j^h = \frac{2M_j\nu}{\pi d_j}. \tag{3}$$

The birth rate can be expressed as following

$$\lambda_j^i = \lambda_j + (p_j + p_3)\lambda_0 + \bar{\lambda}_j^h, i = 0, \dots, C_j - 1, 1 \leq j \leq 2. \tag{4}$$

Similarly the death rate from state i to $i - 1$ is defined as μ_j^i . μ_j^i is made up of two parts: one is the average service completion rate, and the other is the handoff rate from network j to outside of the cell. The current number of users in network j is i and therefore the service completion rate is $i\mu$, According to the handoff rate, which is $2i\nu/(\pi d_j)$, the birth rate can be expressed as

$$\mu_j^i = i \left(\mu + \frac{2\nu}{\pi d_j} \right), i = 1, \dots, C_j, 1 \leq j \leq 2. \tag{5}$$

According to Eq. (2), the steady-state function of network j can be obtained

$$\pi_j^i = \left(\sum_{k=0}^{C_j} \left(\frac{(\xi_j)^k}{k!} \right) \right)^{-1} \frac{(\xi_j)^i}{i!}, 0 \leq i \leq C_j, 1 \leq j \leq 2, \tag{6}$$

where

$$\xi_j = \frac{\lambda_j + (p_j + p_3)\lambda_0 + \bar{\lambda}_j^h}{\mu + 2\nu/(\pi d_j)}. \tag{7}$$

Next based on the conclusion of the steady-state distribution, which is represented as formula (2), the average number of users in network j is calculated

$$\begin{aligned} M_j(\xi_j) &= \sum_{i=0}^{C_j} i\pi_j^i = \xi_j \left(1 - \left(\sum_{k=0}^{C_j} \left(\frac{(\xi_j)^k}{k!} \right) \right)^{-1} \frac{(\xi_j)^{C_j}}{C_j!} \right) \\ &= \xi_j (1 - P_j^b) = \xi_j P_j^{nb}, \end{aligned} \tag{8}$$

where P_j^b and P_j^{nb} denote network block rate and network access rate respectively, and

$$P_j^b = 1 - P_j^{nb} = \pi_j^{C_j} = \left(\sum_{k=0}^{C_j} \left(\frac{(\xi_j)^k}{k!} \right) \right)^{-1} \frac{(\xi_j)^{C_j}}{C_j!} = P(\xi_j). \tag{9}$$

When taking Eq. (8) of average user number into Eq. (3), we can get the average handoff rate of switching from network j to the outside of the cell

$$\overline{\lambda}_j^h = \frac{2M_j(\xi_j)v}{\pi d_j} = \frac{2\xi_j v P_j^{nb}}{\pi d_j} = \frac{2v P_j^{nb}}{\pi d_j} \frac{\lambda_j + (p_j + p_3)\lambda_0 + \overline{\lambda}_j^h}{\mu + 2v/(\pi d_j)}. \quad (10)$$

For the convenience of calculation, the similar terms in Eq. (7) are combined to get the expression about network block rate

$$\overline{\lambda}_j^h = \frac{2v P_j^{nb} (\lambda_j + (p_j + p_N)\lambda_0)}{\pi d_j \mu + 2v P_j^{nb}} = f(P_j^b). \quad (11)$$

Taking the above formula into formula (9), we can get

$$P_j^b = P(\xi_j) \Big|_{\xi_j = \frac{\lambda_j + (p_j + p_N)\lambda_0 + f(P_j^b)}{\mu + 2v/(\pi d_j)}} = g(P_j^b). \quad (12)$$

Here define that

$$\varphi(x) = x - g(x). \quad (13)$$

It is obvious that P_j^b is the root of $\varphi(x) = 0$. Because

$$\varphi'(x) = 1 + \frac{1}{\mu + 2v/(\pi d_j)} \frac{2v(\lambda_j + (p_j + p_N)\lambda_0)(\pi d_j \mu + 2v) P_j^b (C_j - \xi_j P_j^{nb})}{(\pi d_j \mu + 2v)^2} \Big|_{\xi_j = \frac{\lambda_j + (p_j + p_N)\lambda_0 + f(P_j^b)}{\mu + 2v/(\pi d_j)}} \quad (14)$$

The maximum number of users in a network will not be more than the channel number, which is $M_j(\xi_j) = \xi_j P_j^{nb} < C_j \sum_{i=0}^{C_j} \pi_i = C_j$. Therefore we can get $\varphi'(x) > 0$. That is to say, the formula is a monotone increasing function about P_j^b . Besides, $0 < \varphi(x) < 1$, $\varphi(0) = -f(0) < 0$ and $\varphi(1) = 1 - f(1) > 0$, consequently the bisection method can be used for the calculation of P_j^b , which then enables us to get $\overline{\lambda}_j^h$. Acquiring the expression of P_j^b and $\overline{\lambda}_j^h$, we will calculate the steady-state average number of users and system throughput next and then construct the system utility function.

In this paper, the utility function is divided into two parts: The unit price throughput model and network access rate model. Heterogeneous networks aim at satisfying throughput, however we not only consider throughput but also take price into consideration, making the network selection pay more attention to the cost performance of networks. Great throughput is obtained at the cost of high block rate. That is why we take network access rate into account. Specific parameters and models are shown below. The utility function of network j is defined as

$$U(\bar{p}) = \alpha T(\bar{p}) + A(\bar{p}) = \frac{\alpha R_j M(\xi_j)}{Cost_j} + P_j^{nb} = Q_j M(\xi_j) + P_j^{nb}, \quad (15)$$

where $T(\bar{p})$ denotes the unit price throughput model of network j , $A(\bar{p})$ is the network access rate model. α denotes the adjustment factor keeping the two models in the same

order of magnitude. Q_j is set instead of $\frac{\alpha R_j}{Cost_j}$.

The heterogeneous network system model in this paper includes two heterogeneous networks, and the final total system utility is defined as

$$U(\vec{p}) = \sum_{j=1}^{N-1} (Q_j M(\xi_j) - P_j^b) = \sum_{j=1}^{N-1} (Q_j \xi_j (1 - P_j^b) - P_j^b) = \sum_{j=1}^{N-1} \left(Q_j (1 - P_j^b) \frac{\lambda_j + (p_j + p_N) \lambda_0 + \bar{\lambda}_i^h}{\mu + 2\nu / (\pi d_j)} - P_j^b \right) \quad (16)$$

Where N represents dimension of vector \mathbf{p} and $N=3$ here.

To maximize the above utility, a proper vector should be calculated for satisfying network performances. Therefore the above problem can be converted into the following convex optimization problem

$$\begin{aligned} \max U &= \sum_{j=1}^{N-1} (Q_j M(\xi_j) - P_j^b), \\ \text{s.t. } \sum_{j=1}^N p_j &\leq 1, p_1, \dots, p_N \geq 0. \end{aligned} \quad (17)$$

In order to analyze this further, the total system throughput is deduced like this

$$T'(\vec{p}) = \sum_{j=1}^{N-1} (R_j M(\xi_j)) = \sum_{j=1}^{N-1} (Q_j \xi_j (1 - P_j^b)) = \sum_{j=1}^{N-1} \left(Q_j (1 - P_j^b) \frac{\lambda_j + (p_j + p_N) \lambda_0 + \bar{\lambda}_i^h}{\mu + 2\nu / (\pi d_j)} \right). \quad (18)$$

And the system network access rate is as following

$$p_s^{nb} = \frac{M(\xi_1) + M(\xi_2)}{\xi_1 + \xi_2} = \frac{\xi_1 p_1^{nb} + \xi_2 p_2^{nb}}{\xi_1 + \xi_2} \quad (19)$$

3.2 Solution of the Convex Optimization Problem

Appendix A has proven that Eq. (16) is a convex function and the feasible region of Eq. (17) is convex set, therefore Eq. (17) is a convex optimization problem according to its definition. The Lagrange multiplier function expression of the above problem is as follows

$$\begin{aligned} \max L(\vec{p}, \gamma) &= \sum_{j=1}^{N-1} \left(Q_j (1 - P_j^b) \frac{\lambda_j + (p_j + p_N) \lambda_0 + \bar{\lambda}_i^h}{\mu + 2\nu / (\pi d_j)} - P_j^b \right) - \gamma \left(\sum_{j=1}^N p_j - 1 \right), \gamma > 0 \\ \text{s.t. } p_1, \dots, p_N &\geq 0. \end{aligned} \quad (20)$$

Here $N=3$. In accordance with the general method of the Lagrange multiplier function, firstly the partial differentials about p_j ($j=1, 2$) are

$$\frac{\partial L(\vec{p}, \alpha)}{\partial p_j} = \left(Q_j (1 - P_j^b (C_j + 1 - \xi_j P_j^{nb})) - \frac{P_j^b (C_j - \xi_j P_j^{nb})}{\xi_j} \right) \frac{\partial \xi_j}{\partial p_j} - \gamma. \quad (21)$$

$j=3$, the partial differential is

$$\frac{\partial L(\bar{\mathbf{p}}, \alpha)}{\partial p_N} = \sum_{j=1}^{N-1} \left(Q_j (1 - P_j^b (C_j + 1 - \xi_j P_j^{nb})) - \frac{P_j^b (C_j - \xi_j P_j^{nb})}{\xi_j} \right) \frac{\partial \xi_j}{\partial p_j} - \gamma. \quad (22)$$

Among the convex optimization methods, the subgradient method is available for non-differentiable objective functions and needs less storage. It is also simple to solve the convex problems. Therefore the subgradient method is adopted to solve this convex optimization problem [18]. γ can be updated following Eq. (23).

$$\gamma^{l+1} = \left[\gamma^l + s^l \left(\sum_{j=1}^N p_j - 1 \right) \right]^+ \quad (23)$$

The conclusion of $\frac{\partial^2 T}{\partial p_j^2} \leq 0$ from appendix A illustrates that Eqs. (21) and (22) are decreasing functions about \mathbf{p} , and therefore dichotomy is used to solve the following equation.

$$\frac{\partial L(\bar{\mathbf{p}}, \alpha)}{\partial \bar{\mathbf{p}}} = \left[\frac{\partial L(\bar{\mathbf{p}}, \alpha)}{\partial p_1}, \dots, \frac{\partial L(\bar{\mathbf{p}}, \alpha)}{\partial p_N} \right]^T = \bar{\mathbf{0}} \quad (24)$$

Suppose that the solution of Eq. (24) is $\bar{\mathbf{p}}'$, where $\bar{\mathbf{p}}' = (p'_1, \dots, p'_N)$. If there is no zero, we will conduct iterative processing as follows: if $\frac{\partial L(\bar{\mathbf{p}}, \alpha)}{\partial p_j} > 0$ for any $p_j \in [0, 1]$, it means γ is too small. So set $p'_j = 0$ to increase γ . Otherwise set $p'_j = 1$.

Fig. 4 displays a flow chart showing how the subgradient method is used to solve the convex optimization problem. According to it, the specific steps are as follows.

Step 1: Construct the system utility function. Determine the feasible domain of optimization factors and prove the problem is a convex optimization one.

Step 2: Initialize the optimization factor following $\bar{\mathbf{p}}^{(0)} = (p_1^{(0)}, p_2^{(0)}, p_3^{(0)}) = (0, 0, 0)$. Calculate the steady-state distribution of each network.

Step 3: Based on the conclusion of steady-state distribution, deduce γ 's initial value $\gamma^{(0)}$, i.e.,

$$\gamma^{(0)} = \frac{2}{N} \sum_{j=1}^{N-1} \left(Q_j (1 - P_j^b (C_j + 1 - \xi_j P_j^{nb})) - \frac{P_j^b (C_j - \xi_j P_j^{nb})}{\xi_j} \right) \frac{\partial \xi_j}{\partial p_j}, \quad (25)$$

Where N denotes the dimension of factor vector and $N=3$ here. Now $l=0$.

Step 4: Follow Eq. (22), use dichotomy to calculate $\bar{\mathbf{p}}'$ and update $\bar{\mathbf{p}}^{(l+1)}$ according to iterative processing methods.

Step 5: According to the judgment conditions, determine whether to stop the iteration. If

stop, jump to Step 6. Otherwise update γ as Eq. (23), and then go to Step 3.

Step 6: Obtain rate factor vector \bar{p} and $\bar{p} = \bar{p}^{(l+1)}$ in this step. CRRM servers send decision information to RRM entities responsible for specific resource allocation.

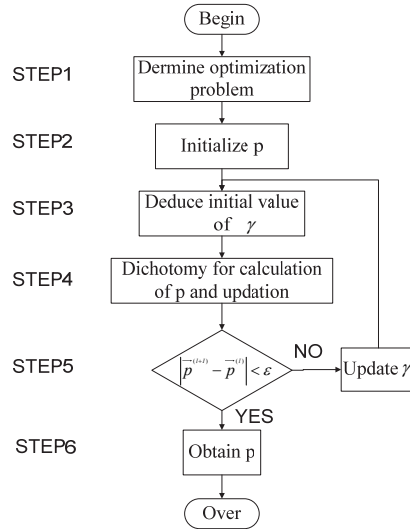


Fig. 4. Flow chart of optimization method.

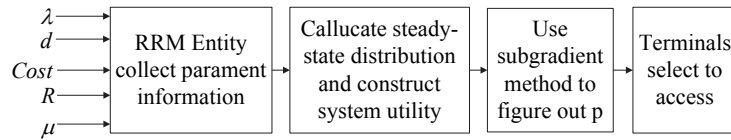


Fig. 5. Block diagram of optimization method.

In summary the block diagram of the birth and death enhanced optimization method for multi-access selection in heterogeneous network is shown in Fig. 5. From the block diagram, RRM entities corresponding to each network are responsible for collecting network parameter information, including the terminal mobility speed, the network coverage radius, the bandwidth of each channel, the number of channels and the traffic arrival rate, and then sending them to the CRRM servers. Based on the birth and death process theory, CRRM servers conduct network selection decision-making according to received information and send them to RRM entities as feedback. Normally CRRM servers calculate the steady-state distribution to construct system utility, and then Fig. out the proportions of different network selections to maximize system utility. The constructed utility problem is modeled as a convex optimization problem and the subgradient method is used to solve this problem.

Allocating radio resources following rate factor will maximize system utility. Under the premise of guaranteed rate factor, RRM entities permit mobile terminals to select networks in freedom, guaranteeing system performance and offering autonomy to users to meet their own requirements.

4. SIMULATION ANALYSIS

The system model of heterogeneous network is shown in Fig. 1. The heterogeneous scenario consists of two heterogeneous networks. We define that the two RANs are UMTS (Universal Mobile Telecommunications System) and WiMAX (Worldwide Interoperability for Microwave Access) respectively. The two heterogeneous networks have an overlapping area. It is supposed that all channels provided by them are serving video traffic. Network parameters are given in table 1 in detail. In this paper, MATLAB software is used for simulation. The simulation environment programming by MATLAB is established according to the settings of WiMAX and UMTS networks [19], and it is the approximate case of real environment, in which the result is approximation of theoretical results. We focus on the performance evaluation, and evaluate the performance of heterogeneous wireless networks by adopting the birth and death process, which is a frequent way to study the performance of network [20]. The framework of the birth and death process is determined by the parameters of heterogeneous wireless networks, which will affect the steady probabilities. As the relationship between λ_0 and other traffic arrival rates, we should adjust the traffic arrival rates with the change of overlap area in the system model. For convenience, we keep λ_1 and λ_2 , and adjust λ_0 to adapt to the relative size between them. Also, it should be noted that the WiMAX network has a higher data rate than the UMTS network, but it has a disadvantage at price because it has not been as well promoted commercially as UMTS network. According to Eq. (16) and the parameters in Table 1, $Q_1=0.192$ and $Q_2=0.212$, showing that the WiMAX network has a more satisfying cost performance.

Table 1. Heterogeneous network parameters.

Parameter	UMTS	WiMAX
$d_i(\text{m})$	500	800
C_i	12	15
$R_i(\text{Mbps})$	0.096	0.136
$\lambda_i(\text{calls/s})$	3/60	4/60
$1/\mu(\text{s/call})$		120
$v(\text{m/s})$		5,10
$\lambda_0(\text{calls/s})$		3/120~7/120
$Cost(\text{RMB/s})$	0.0050	0.0064
α		0.01

4.1 Explanation of Upper and Lower Bounds

Fig. 6 illustrates the comparison of the UMTS network block rate and upper and lower bounds of the network block rate, while parameters are given in Table 1. The upper and lower bounds of the network block rate are given as formula (A.10) and (A.11) respectively. Suppose that there are 12 video channels in UMTS network. Based on the following simulation results, the network block rate is no more than 20%. As a result of this and formula (8), when ξ_1 ranges from 5 to 15, it includes every situation that channels are used. Similarly Fig. 7 shows the comparison of the WiMAX network block rate

and the upper and lower bounds of the network block rate while parameters are given in table 1. The number of video channels is 15 and the block rate is similar to that of the UMTS network. Therefore the range of ξ_2 from 10 to 20 also covers every case. As shown in Figs. 6 and 7, the block rates of the two heterogeneous networks both lie between the upper and lower bounds, guaranteeing the utility problem is a convex optimization one.

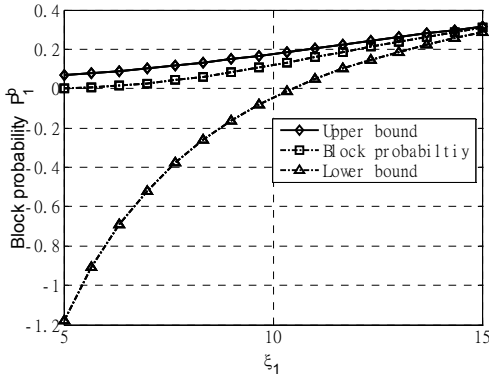


Fig. 6. Comparison of UMTS network block rate and bounds.

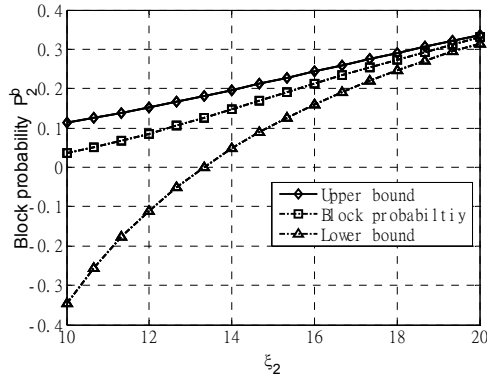


Fig. 7. Comparison of WiMAX network block rate and bounds.

4.2 Simulation Results

Figs. 8 to 18 demonstrate the simulation results. In order to fully show the characteristics of the algorithm in this paper, the algorithm setting rate factor vector $\bar{\mathbf{p}} = [p_1 \ p_2 \ p_3] = \bar{\mathbf{p}}_0 = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$, the single network selection algorithm, of which every terminal accesses only one network in the overlapping area, and the CAVSH algorithm in [10], are adopted for comparison. Specifically $\bar{\mathbf{p}}_0 = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$ represents the average state of random distribution; The sum of p_1 and p_2 equals one, simplifying the optimization algorithm as a one-dimensional problem, which can be solved by the golden section method; The CAVSH algorithm is a multi-access networks selection method. According to the number of users in active set and the total number of users, we can calculate the rates of multi-access and single-access.

Figs. 8-10 show how the three proportions of different network selections change with λ_0 . Additionally p_1 is the proportion of access to UMTS network, p_2 is that of WiMAX and p_3 represents the multi-access rate. It can be seen from the three Figs that when the traffic arrival rate is low, all terminals tend to access to both networks simultaneously for a more satisfying video service experience. With the traffic arrival rate increases, the proportions of p_1 and p_2 keep increasing while p_3 decreases, showing that the larger traffic arrival rate results in the decline of unoccupied channels and more fierce competition among users, making the proportions of the single network selection become larger and that of multi-access is reduced. Comparing Fig. 8 with Fig. 9, the tendency of

proportion access to the UMTS network is similar to that of the WiMAX network, but p_2 stays larger than p_1 , illustrating that users are more inclined to access to the WiMAX network because of a better cost performance. Besides Figs. 8-10 also give proportions in a different velocity, namely $v=5\text{m/s}$ and $v=10\text{m/s}$. The tendencies of proportions changing of the two speeds are similar. However a greater handoff rate resulted by the higher speed means a larger probability of users' switching out and smaller number of users left in the cell. Consequently p_3 of $v=10\text{m/s}$ is larger than that of $v=5\text{m/s}$, and p_1 and p_2 of $v=10\text{m/s}$ are smaller. p_1, p_2 and p_3 consist of the rate factor vector \mathbf{p} , and it is noted that the sum of the three rates of heterogeneous network overlapping area keeps 1 when λ_0 changes thereby guaranteeing the convergence and effectiveness of the proposed algorithm.

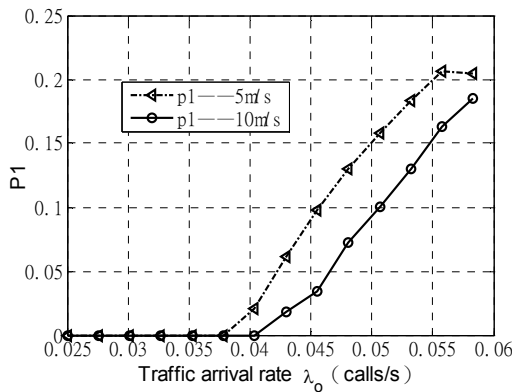


Fig. 8. Rate of access to UMTS network in heterogeneous network overlapping area.

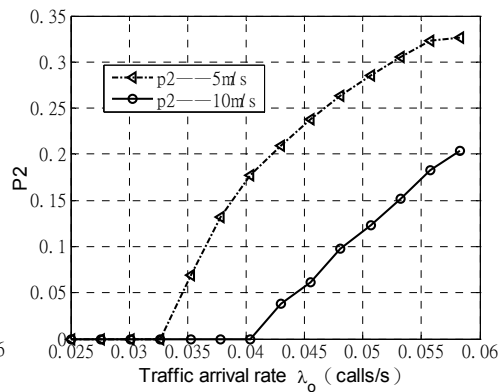


Fig. 9. Rate of access to WiMAX network in heterogeneous network overlapping area.

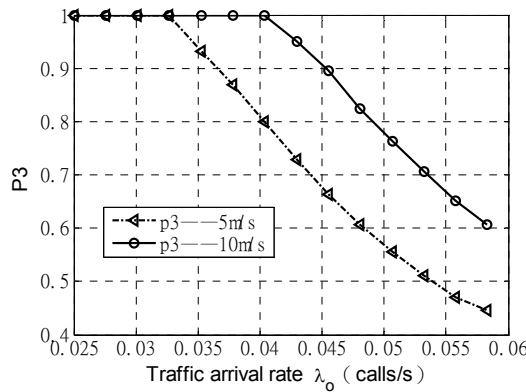


Fig. 10. Rate of multi-access in heterogeneous network overlapping area.

Figs. 11-12 illustrate how the heterogeneous network system utilities with different speeds change with the increase of λ_0 . Figs. 11 and 12 show the tendency of $v=5\text{m/s}$ and $v=10\text{m/s}$ respectively. According to these figures, the utilities of the proposed algorithm are better than that of $\vec{p}_0 = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$, the single network selection algorithm and the

CAVSH method, demonstrating that the adoption of multi-access selection algorithm in the heterogeneous network overlapping region improves the effectiveness of radio resources. Statistically it can be seen that when $v=5\text{m/s}$, the proposed method has a 1.0% to 6.2% improvement compared to that of \bar{p}_0 and has a 4.9% to 10.2% improvement compared to the single network selection method on system utility. When $v=10\text{m/s}$, the improvements ratios are from 2.7% to 6.1% and from 6.0% to 10.7% respectively. A larger traffic rate in the overlapping area leads to a larger average number of users and that is why the system utilities of the four comparing methods all raise when traffic arrival rate in the overlapping area increases. However the method of $\bar{p}_0 = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$ shares a constant rate vector resulting in a nearly linear increase in utilities, which is similar to that of the single network selection algorithm. The CAVSH method has a similar tendency of utilities to our method. However it does not allocation radio resources for maximizing the system utility and that is why its utilities are lower. To maximize the system utility of the proposed algorithm in this paper, the average number of users in a cell grows slowly because of the limited number of channels and multi-access selection, resulting in the slow growth of utility. The tendency of utility of the proposed method conforms to the actual situation.

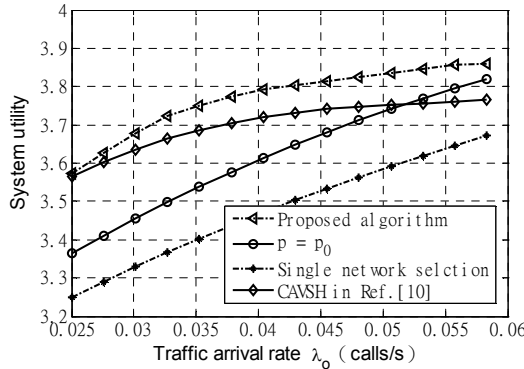


Fig. 11. Utilities of $v=5\text{m/s}$ with different traffic arrival rate.

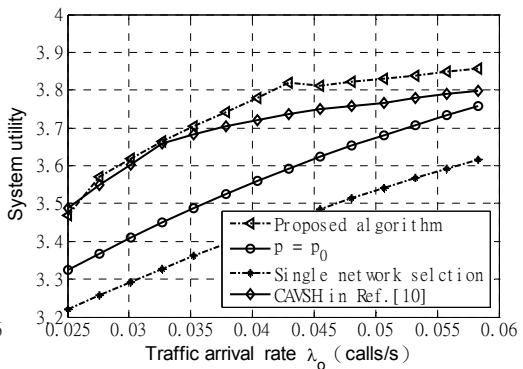


Fig. 12. Utilities of $v=10\text{m/s}$ with different traffic arrival rate.

Figs. 13 and 14 illustrate how the heterogeneous network system throughputs with different speeds change with the increase of λ_0 . Figs. 13 and 14 show the tendency of $v=5\text{m/s}$ and $v=10\text{m/s}$ respectively. The expression of system throughput is given in formula (18). Comparing the tendency of throughput and utility, they change similarly. According to the two figure it can be seen that not only when $v=5\text{m/s}$ but also $v=10\text{m/s}$, the throughputs of the proposed algorithm are better than that of $\bar{p}_0 = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$, the single network selection algorithm and the CAVSH method. This illustrates that the use of the multi-access selection algorithm in the heterogeneous network overlapping area improves the efficiency of the radio resources and contributes to a more satisfying performance of throughput, which is one optimization goal of heterogeneous network. Statistically it can be seen that when $v=5\text{m/s}$ the proposed method has a 1.7% to 8.0% improve-

ment compared to that of \bar{p}_0 and has a 7.2% to 10.7% improvement compared to the single network selection method on the system throughput. When $v=10\text{m/s}$, the improvement ratios are from 3.5% to 8.6% and from 8.2% to 12.0% respectively. Additionally the increase of the traffic rate in the overlapping area leads to the growth of the average number of users in a cell, which could be a reason why system throughput rises with the increase of λ_0 . In contrast the method of $\vec{p}_0 = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$ shares a constant rate vector, resulting in a linear increase in throughput, which is similar to that of the single network selection algorithm. The CAVSH method has a similar tendency of throughputs to our method. However the greed method used for throughput allocation does not work perfectly and that is why its throughputs are lower. To maximize the system utility of the proposed algorithm in this paper, the average number of users in a cell grows slowly because of the limited number of channels and multi-access selection, resulting in the slow growth of throughput, which is similar to that of utility.

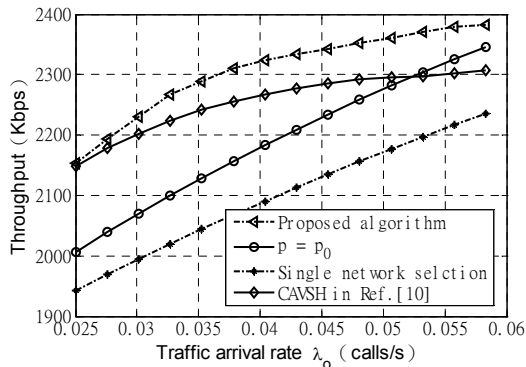


Fig. 13. Throughputs of $v=5\text{m/s}$ with different traffic arrival rate.

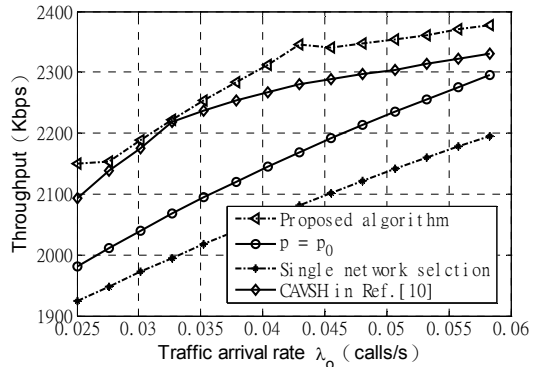


Fig. 14. Throughputs of $v=10\text{m/s}$ with different traffic arrival rate.

Figs. 15-16 illustrate how the network access rate (network non-block rate) with different speeds changes with the increase of λ_0 . Figs. 15 and 16 show the system network access rates when $v=5\text{m/s}$ and $v=10\text{m/s}$ respectively. According to Fig. 15, the network access rates of the four methods are reduced when λ_0 grows. A bigger λ_0 leads to a larger average number of users which result in more fierce competition among users and a worse network access rate. The proposed method does not work well on network access rate when $v=5\text{m/s}$.

Additionally the network access rate gap between the proposed algorithm and method of \bar{p}_0 is narrowed when traffic rate grows. In Fig. 15, when λ_0 becomes 0.05 calls/s, there is no difference in the network access rate between the two methods. When λ_0 becomes larger than 0.05 call/s, the proposed method outweighs the algorithm \bar{p}_0 . According to Figs. 8-10, when λ_0 becomes 0.05 calls/s, the rate of multi-access selection is very small, meaning there is a larger average number of users in the heterogeneous network system and a more satisfying performance of network access rate. The CAVSH has a better performance here because of the lower rate of multi-access. It is obvious that the change of network access rate of the proposed algorithm conforms to the actual situation.

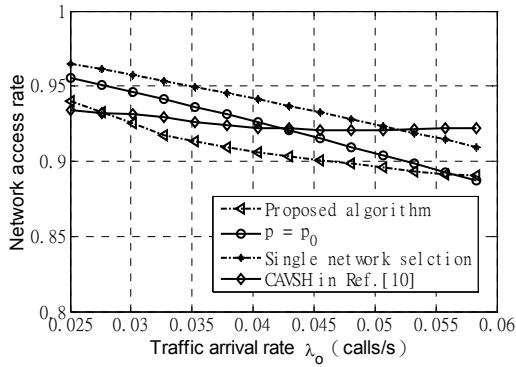


Fig. 15. System network access rate when $v = 5\text{m/s}$ with different traffic arrival rate.

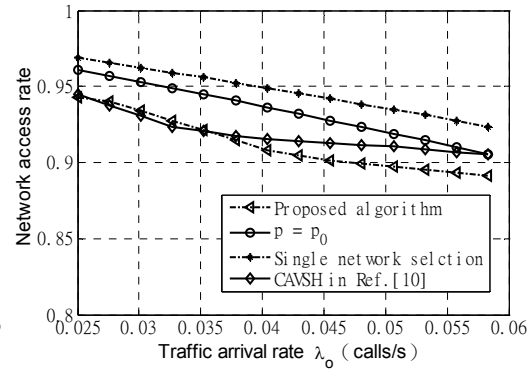


Fig. 16. System network access rate when $v = 10\text{m/s}$ with different traffic arrival rate.

System network access rate of $v=10\text{m/s}$ shares a similar performance to that of $v=5\text{m/s}$. When $v=10\text{m/s}$, it also has a disadvantage in the network access rate compared to the method of $\bar{p}_0 = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$ and single network selection algorithm as a result of a big rate of multi-access selection. Specifically it has similar network access rate performance with the CAVSH method. According to Fig. 16, the network access rates of the four methods decrease when λ_0 grows. When λ_0 becomes larger, there are more users in a cell, which means more fierce competition among users and worse network access rate. Similarly the access rate of $v=10\text{m/s}$ is worse than that of method of \bar{p}_0 and the single network selection algorithm, but it also keeps more than 85%. Additionally the system network access rate also gets close to that of method of \bar{p}_0 .

Above all the proposed optimization algorithm for multi-access selection in heterogeneous wireless networks maximizes system utility, has satisfying performances of throughput and network access rate, and enhances the efficiency of radio resources. However the heterogeneous network scenario in this paper is relatively simple. Although the utility function is a multi-target problem which can provide satisfying QoS, the network access rate should be improved and the solution to the optimization problem should be more simplified and more innovative. These problems will be solved in future research.

5. CONCLUSION

In this paper, a Markov process based utility optimization method for multi-access selection in heterogeneous wireless networks is proposed. The method adopts continuous time Markov process: the birth and death process to analyze the system performance of heterogeneous networks. Also the method supports users' mobility and uses handoff rate to replace mobility. Then the system utility function is obtained. The proportions of different multi-access selection in the heterogeneous wireless network overlapping region should be well organized in order to maximize the system utility. The subgradient method is adopted for solving this optimization problem. The simulation results show that the method can maximize system utility, effectively improve throughput and make the radio resource allocation more efficient.

APPENDIX A: Proof of system utility being a convex optimization problem

Eq. (16) need to be proven as a convex function to illustrate that Eq. (17) is a convex optimization problem. Here we can prove that the utilities' second derivative matrix (Hessian Matrix) about vector \mathbf{p} , which is set as matrix \mathbf{A} , is negative definite to illustrate the above problem. Based on the conclusion of the convex optimization problem, *i.e.* the sufficient and necessary condition for the matrix \mathbf{A} is negative definite is matrix $-\mathbf{A}$ is positive definite, we can judge $-\frac{\partial^2 U}{\partial \mathbf{p}^2}$ is positive definite to prove the system utility is a convex optimization problem. The Hessian Matrix $-\frac{\partial^2 U}{\partial \mathbf{p}^2}$ is as following

$$-\mathbf{A} = -\frac{\partial^2 U}{\partial \mathbf{p}^2} = \begin{bmatrix} -\frac{\partial^2 U}{\partial p_1^2} & 0 & -\frac{\partial^2 U}{\partial p_1 \partial p_3} \\ 0 & -\frac{\partial^2 U}{\partial p_2^2} & -\frac{\partial^2 U}{\partial p_2 \partial p_3} \\ -\frac{\partial^2 U}{\partial p_3 \partial p_1} & -\frac{\partial^2 U}{\partial p_3 \partial p_2} & -\frac{\partial^2 U}{\partial p_3^2} \end{bmatrix}. \quad (\text{A.1})$$

It is easy to find that $\frac{\partial^2 U}{\partial p_3 \partial p_1} = \frac{\partial^2 U}{\partial p_1 \partial p_3} = \frac{\partial^2 U}{\partial p_1^2}$ and $\frac{\partial^2 U}{\partial p_3 \partial p_2} = \frac{\partial^2 U}{\partial p_2 \partial p_3} = \frac{\partial^2 U}{\partial p_2^2}$, therefore the above matrix can be converted into

$$-\mathbf{A} = -\frac{\partial^2 U}{\partial \mathbf{p}^2} = \begin{bmatrix} -\frac{\partial^2 U}{\partial p_1^2} & 0 & -\frac{\partial^2 U}{\partial p_1^2} \\ 0 & -\frac{\partial^2 U}{\partial p_2^2} & -\frac{\partial^2 U}{\partial p_2^2} \\ -\frac{\partial^2 U}{\partial p_1^2} & -\frac{\partial^2 U}{\partial p_2^2} & -\frac{\partial^2 U}{\partial p_3^2} \end{bmatrix}. \quad (\text{A.2})$$

It is apparent that matrix (A.2) is a third order real symmetric matrix. According to the lemma in literature [15], *i.e.* the sufficient and necessary condition that n order real symmetric matrix is half positive definite is that all the eigenvalues of the matrix are nonnegative, and the formula $\frac{\partial^2 U}{\partial p_3^2} = \frac{\partial^2 U}{\partial p_1^2} + \frac{\partial^2 U}{\partial p_2^2}$, then the row's linear transformation to matrix is used to obtain

$$-\mathbf{A}' = \begin{bmatrix} -\frac{\partial^2 U}{\partial p_1^2} & 0 & -\frac{\partial^2 U}{\partial p_1^2} \\ 0 & -\frac{\partial^2 U}{\partial p_2^2} & -\frac{\partial^2 U}{\partial p_2^2} \\ 0 & 0 & 0 \end{bmatrix}. \quad (\text{A.3})$$

The eigenvalues of matrix \mathbf{A} and $-\mathbf{A}$ are the same. Next we will figure out the eigenvalues of matrix $-\mathbf{A}$. According to the definition of eigenvalues, they are the solutions of below equation.

$$|-\mathbf{A}' - \lambda \mathbf{E}| = 0, \quad (\text{A.4})$$

Where \mathbf{E} is a third-order unit matrix and the eigenvalues are calculated as following

$$\begin{cases} \lambda_1 = -\frac{\partial^2 U}{\partial p_1^2} \\ \lambda_2 = -\frac{\partial^2 U}{\partial p_2^2} \\ \lambda_3 = 0 \end{cases} \quad (\text{A.5})$$

Therefore, λ_1 , λ_2 and λ_3 should be all nonnegative, that is

$$\begin{cases} \frac{\partial^2 U}{\partial p_1^2} \leq 0 \\ \frac{\partial^2 U}{\partial p_2^2} \leq 0 \end{cases} \quad (\text{A.6})$$

Matrix $-\mathbf{A}$ is half positive definite [16]. The above conditions are enough to prove that Eq. (16) is a convex function.

Additionally for network $j(j=1, 2)$, the proof of the condition (A.6) is shown as following

$$\frac{\partial P_j^b}{\partial \xi_j} = \frac{P_j^b (C_j - \xi_j P_j^{nb})}{\xi_j}, \quad (\text{A.7})$$

$$\frac{\partial U}{\partial \xi_j} = Q_j (1 - P_j^b (C_j + 1 - \xi_j P_j^{nb})) - \frac{P_j^b (C_j - \xi_j P_j^{nb})}{\xi_j}, \quad (\text{A.8})$$

$$\begin{aligned} \frac{\partial^2 U}{\partial \xi_j^2} &= -Q_j \frac{P_j^b}{\xi_j} ((C_j + 1 - \xi_j P_j^{nb})(C_j - \xi_j P_j^{nb} + \xi_j P_j^b) - \xi_j) \\ &\quad - \frac{P_j^b}{\xi_j^2} (C_j - \xi_j + 2\xi_j P_j^b)(C_j - \xi_j P_j^{nb}) + \frac{C_j P_j^b}{\xi_j^2}. \end{aligned} \quad (\text{A.9})$$

Set $\frac{\partial U}{\partial \xi_j} \geq 0$, then

$$P_j^b \leq \frac{Q_j}{Q_j(C_j + 1 - \xi_j P_j^{nb}) + (C_j - \xi_j P_j^{nb})/\xi_j}. \quad (\text{A.10})$$

Suppose formula (A.10) as the upper bound of network block rate.

Set $\frac{\partial^2 U}{\partial \xi_j^2} \leq 0$, then

$$P_j^b \geq \frac{C_j + Q_j \xi_j^2}{\xi_j (Q_j \xi_j (C_j + 1 - \xi_j P_j^{nb}) + (C_j - \xi_j P_j^{nb}))} - \frac{C_j - \xi_j P_j^{nb}}{\xi_j}. \quad (\text{A.11})$$

Suppose formula (A.11) as the lower bound of network block rate. Bounds should be met to get the optimal solution of the optimization problem.

The first and second partial derivatives about vector \mathbf{p} of utility U are

$$\frac{\partial U}{\partial p_j} = \frac{\partial U}{\partial \xi_j} \frac{\partial \xi_j}{\partial p_j}, \quad (\text{A.12})$$

$$\frac{\partial^2 U}{\partial p_j^2} = \frac{\partial^2 U}{\partial \xi_j^2} \left(\frac{\partial \xi_j}{\partial p_j} \right)^2 + \frac{\partial U}{\partial \xi_j} \frac{\partial^2 \xi_j}{\partial p_j^2}. \quad (\text{A.13})$$

Based on the conclusions in literature [17]

$$\frac{\partial \xi_j}{\partial p_j} = \frac{\lambda_0}{\mu + 2v/(\pi d_j)} \left(1 + \frac{2vP_j^{nb}}{\pi d_j \mu + 2vP_j^b} \right) \left(1 + \frac{2\pi P_j^b d_j v (\lambda_j + (p_j + p_N) \lambda_0 (C_j - \xi_j P_j^{nb}))}{(\pi d_j \mu + 2vP_j^b)^2 \xi_j} \right)^{-1} \quad (\text{A.14})$$

It is obvious that $\frac{\partial \xi_j}{\partial p_j} > 0$.

$$\text{As } \frac{\partial^2 \xi_j}{\partial p_j^2} = \frac{2v/(\pi d_j)}{\mu + 2v/(\pi d_j)} \frac{\partial^2 M(\xi_j)}{\partial \xi_j^2} \left(\frac{\partial \xi_j}{\partial p_j} \right)^2 \left(1 - \frac{2v/(\pi d_j)}{\mu + 2v/(\pi d_j)} \frac{\partial M(\xi_j)}{\partial \xi_j} \right)^{-1} \quad (\text{A.15})$$

Considering

$$\frac{\partial M(\xi_j)}{\partial \xi_j} = \frac{\partial (\xi_j (1 - P_j^b))}{\partial \xi_j} = 1 - P_j^b - \xi_j \frac{\partial P_j^b}{\partial \xi_j} = 1 - P_j^b (C_j + 1 - \xi_j P_j^{nb}). \quad (\text{A.16})$$

According to upper limit in [17], that is $P_j^b < \frac{1}{C_j + 1 - \xi_j P_j^{nb}}$ we can find

$$0 < \frac{\partial M(\xi_j)}{\partial \xi_j} < 1, \text{ then } 0 < 1 - \frac{2v/(\pi d_j)}{\mu + 2v/(\pi d_j)} \frac{\partial M(\xi_j)}{\partial \xi_j} < 1.$$

$$\text{Because } \frac{\partial^2 M(\xi_j)}{\partial \xi_j^2} = -\frac{P_j^b}{\xi_j} \left((C_j + 1 - \xi_j P_j^{nb})(C_j - \xi_j P_j^{nb} + \xi_j P_j^b) - \xi_j \right). \quad (\text{A.17})$$

According to lower limit in [17], that is $P_j^b > \frac{1}{C_j + 1 - \xi_j P_j^{nb}} - \frac{C_j - \xi_j P_j^{nb}}{\xi_j}$ it is easy to find

$$\frac{\partial^2 M(\xi_j)}{\partial \xi_j^2} < 0, \text{ therefore } \frac{\partial^2 \xi_j}{\partial p_j^2} < 0.$$

The condition that both bounds are met, i.e. $\frac{\partial U}{\partial \xi_j} \geq 0$ and $\frac{\partial^2 U}{\partial \xi_j^2} \leq 0$, and $\frac{\partial^2 \xi_j}{\partial p_j^2} < 0$, then $\frac{\partial^2 U}{\partial p_j^2} \leq 0$ ($j=1, 2$). Consequently matrix A is half negative definite and formula (16) is a convex function.

REFERENCES

1. L. J. Wu and K. Sandrasegaran, "A survey on common radio resource management, wireless broadband and ultra wideband communications," in *Proceedings of the 2nd International Conference on Aus Wireless*, 2007, pp. 66-71.
2. W. X. Shi, S. S. Fan, N. Wang, and C. J. Xia, "Fuzzy neural network based access selection algorithm in heterogeneous wireless networks," *Journal on Communications*, Vol. 31, 2010, pp. 151-156.
3. Y. Y. Zhang, Hang Long, F. Liu, W. Wang, and L. Lei, "A QoE-aware method for energy efficient network selection," in *Proceedings of International Symposium on Communications and Information Technologies*, 2012, pp. 850-854.
4. A. Sgora, D. D. Vergados, and P. Chatzimisios, "An access network selection algorithm for heterogeneous wireless environment," in *Proceedings of IEEE Symposium on Computers and Communications*, 2010, pp. 890-892.
5. A. Hasib and A. O. Fapojuwo, "Analysis of common radio resource management scheme for end-to-end QoS support in multiservice heterogeneous wireless networks," *IEEE Transactions on Vehicular Technology*, Vol. 57, 2008, pp. 2426-2439.
6. X. L. Ai, W. N. Zhou, B. Xie, and J. Song, "Network selection issue in heterogeneous wireless environment," in *Proceedings of IEEE Wireless Communications and Networking Conference*, 2010, pp. 1-6.
7. S. K. Lee, K. S. K. Kim, J. H. Lee, Y. H. Kim, and N. Golmie, "Vertical handoff decision algorithm providing optimized performance in heterogeneous wireless networks," *IEEE Transactions on Vehicular Technology*, Vol. 58, 2008, pp. 865-881.
8. W. H. Fan, Y. N. Liu, and F. Wu, "Research on multi-access selection mechanism for multi-mode terminals in heterogeneous wireless networks," *Journal of Communications*, Vol. 33, 2012, pp. 183-190.
9. L. N. Zhang and Q. Zhu, "Multi-access selection algorithm based on joint utility optimization for the fusion of heterogeneous wireless networks," *IEICE Transactions on Communications*, Vol. E97-B(1), 2014, pp. 2269-2277.
10. K. M. Yang, "Context aware vertical soft handoff algorithm for heterogeneous wire-

- less networks,” *IEEE Vehicular Technology Conference*, 2008, pp. 1-5.
11. P. R. Jorpez and O. Sallen, “Loose and tight interworking between vertical and horizontal handovers in multi-RAT scenarios,” in *Proceedings of IEEE Mediterranean Electrotechnical Conference*, 2006, pp. 579-582.
 12. C. H. Liu, *Stochastic Process With Its Applications*, 3rd ed., Higher Education Press, 2013, pp. 98-103.
 13. Z. Shi and Q. Zhu, “A vertical handoff rate analysis based on angle mobility model in heterogeneous networks,” *Signal Processing*, Vol. 28, 2012, pp. 1029-1036.
 14. L. S. Wang and G. S. Kuo, “Mathematical modeling for network selection in heterogeneous wireless networks,” *IEEE Communications Surveys and Tutorials*, Vol. 15, 2013, pp. 271-292.
 15. H. M. Wei, “Promotion on the nature of positive semi-definite quadratic form and positive semi-definite matrix,” *Journal of Chifeng University*, 2012, pp. 3-5.
 16. K. X. Xie, J. Han, and Y. L. Lin, *Optimization Method*, Tianjin University Press, 2013, pp. 28-40.
 17. Z. Shi, “Research on the technology of radio resource management in heterogeneous wireless networks,” Nanjing University of Posts and Telecommunications, 2013, pp. 47-59.
 18. W. Yu and R. Lui, “Dual methods for nonconvex spectrum optimization of multi-carrier systems,” *IEEE Transactions on Communications*, Vol. 54, 2006, pp. 1310-1322.
 19. Z. Shi, “Performance analysis of an improved common radio resource management algorithm based on load balance and service characteristics,” *Wireless Personal Communications*, Vol. 71, 2013, pp. 317-338.
 20. T. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed., Englewood Cliffs, NJ, Prentice-Hall, 2002, pp. 555-564.



Shuo Zhang (张硕) was born in Jiangsu, China in December, 1990. He is working toward the M.S. degrees in Communication and Information System at Nanjing University of Posts and Telecommunications and in Computer Science at University of Stavanger, Norway. His research interests include networks selection in heterogeneous wireless network, vertical handoff and radio resource management.



Qi Zhu (朱琦) received the M.S. degree in Radio Engineering from Nanjing University of Posts and Telecommunications in 1989. Now she is a Professor in the Department of Telecommunication and Information Engineering, Nanjing University of Posts and Telecommunications, Jiangsu, China. Her research interests include radio resource management, heterogeneous networks, and green communication.